



Determination of Wind Turbine Near-Wake Length Based on Stability Analysis

Sørensen, Jens Nørkær; Mikkelsen, Robert Flemming; Sarmast, Sasan; Ivanell, Stefan; Henningson, Dan

Published in:
Journal of Physics: Conference Series (Online)

Link to article, DOI:
[10.1088/1742-6596/524/1/012155](https://doi.org/10.1088/1742-6596/524/1/012155)

Publication date:
2014

Document Version
Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

Citation (APA):
Sørensen, J. N., Mikkelsen, R. F., Sarmast, S., Ivanell, S., & Henningson, D. (2014). Determination of Wind Turbine Near-Wake Length Based on Stability Analysis. *Journal of Physics: Conference Series (Online)*, 524(1), [012155]. <https://doi.org/10.1088/1742-6596/524/1/012155>

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Determination of Wind Turbine Near-Wake Length Based on Stability Analysis

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2014 J. Phys.: Conf. Ser. 524 012155

(<http://iopscience.iop.org/1742-6596/524/1/012155>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 192.38.90.17

This content was downloaded on 20/06/2014 at 10:33

Please note that [terms and conditions apply](#).

Determination of Wind Turbine Near-Wake Length Based on Stability Analysis

Jens N. Sørensen^{1,2}, Robert Mikkelsen¹, Sasan Sarmast^{1,3}, Stefan Ivanell^{2,3}
and Dan Henningson³

¹DTU Wind Energy, Technical University of Denmark, 2800 Lyngby, Denmark

²Uppsala University, Dep. of Earth Sciences, Campus Gotland, Visby, Sweden

³Linné FLOW Centre, KTH Mechanics, Stockholm, Sweden

Email: jnso@dtu.dk

Abstract: A numerical study on the wake behind a wind turbine is carried out focusing on determining the length of the near-wake based on the instability onset of the trailing tip vortices shed from the turbine blades. The numerical model is based on large-eddy simulations (LES) of the Navier-Stokes equations using the actuator line (ACL) method. The wake is perturbed by applying stochastic or harmonic excitations in the neighborhood of the tips of the blades. The flow field is then analyzed to obtain the stability properties of the tip vortices in the wake of the wind turbine. As a main outcome of the study it is found that the amplification of specific waves (traveling structures) along the tip vortex spirals is responsible for triggering the instability leading to wake breakdown. The presence of unstable modes in the wake is related to the mutual inductance (vortex pairing) instability where there is an out-of-phase displacement of successive helix turns. Furthermore, using the non-dimensional growth rate, it is found that the pairing instability has a universal growth rate equal to $\pi/2$. Using this relationship, and the assumption that breakdown to turbulence occurs once a vortex has experienced sufficient growth, we provide an analytical relationship between the turbulence intensity and the stable wake length. The analysis leads to a simple expression for determining the length of the near wake. This expression shows that the near wake length is inversely proportional to thrust, tip speed ratio and the logarithmic of the turbulence intensity.

1. Introduction

Modern wind turbines are often clustered in wind farms and, depending on the wind direction, the turbines are fully or partially influenced by the upstream turbine wakes. Hence, an unwanted but inevitable effect is that the efficiency of the interior turbines decreases due to velocity deficits and the turbulence intensity increases due to the interaction from the wakes of the surrounding wind turbines. As a consequence, dynamic loadings increase, which may excite the resonance frequency in the structural parts of the individual wind turbines and increase the fatigue loads. The turbulence created from wind turbine wakes is mainly due to the presence of the distinct tip and root vortices. In most of the situations, the organized tip/root vortex system is unstable (Okulov and Sørensen, 2007) and it eventually breaks down and forms small scale turbulent structures. It is important to note that if a wind turbine is located in a wake consisting of stable tip and root vortices, the fatigue loading is more severe than in the case where the tip vortices have already broken down by instability mechanisms (Sørensen 2011). Understanding the physical nature of the vortices and their dynamics in the wake of a turbine is thus important for the optimal design of a wind farm.



The stability of the tip vortices of a wind turbine has been investigated by Ivanell *et al.* (2010) using full CFD computations combined with the actuator line (ACL) technique. In the ACL method, which was developed by Sørensen & Shen (2002), the presence of the blades is introduced as a body force and the flow field around the blades is determined by solving the three-dimensional Navier–Stokes equations using large-eddy simulations (LES). The resulting wake was subsequently perturbed by imposing a harmonic excitation near the tip of the blade. Analyzing the flow field indicated that the instability is dispersive and that the spatial growth arises for different frequencies and spatial structures, the strongest being wave numbers equal to half-integer multiples of the number of blades, as was previously found in the inviscid investigations by e.g. Gupta & Loewy (1974), Bhagwat & Leishman (2001) and Leishman *et al.* (2004).

The pairing instability in the wake of a wind turbine was first seen in the smoke visualization of Alfredsson & Dahlberg (1979). Recent experiments by Felli *et al.* (2011) and Leweke *et al.* (2013) as well as numerical studies by Widnall (1972) and Ivanell *et al.* (2010) have confirmed that the mutual inductance instability leads to vortex pairing in the rotor wake, and they indicate that the vortex pairing is the primary cause of wake destabilization. The vortex pairing is a result of the vortex-induced velocities in a form that it is analogous to the leapfrogging motion of two inviscid vortex rings. This phenomenon occurs in a row of equidistant identical vortices, whereby amplifications of small perturbations cause the vortices to oscillate such a way that neighboring vortices approach each other and start to group in pairs. Lamb (1932) has analyzed the stability of single and double rows of identical vortices in two dimensions, representing the parallel helical vortices in three dimensions. He found that the maximum non-dimensional temporal growth rate can reach up to $\sigma = \pi/2$ in both setups. Ivanell *et al.* (2010) and Leweke *et al.* (2013) also reached a similar conclusion that the highest growth rate perturbations have a non-dimensional growth rate close to $\sigma = \pi/2$, albeit in their studies it was the spatial growth rate that was found.

The present study we show how stability theory and knowledge of the growth rate combined with basic rotors aerodynamics can be exploited to determine the length of the near wake behind a wind turbine.

2. Numerical modelling

The actuator line (ACL) method, introduced by Sørensen & Shen (2002), is a fully three-dimensional and unsteady aerodynamic model for simulating wind turbine wakes. In this method, the flow around the rotor is governed by the three-dimensional incompressible Navier–Stokes equations using large eddy simulation technique, while the influence of the blades on the flow field is approximated by a body force. The force is determined using a blade-element approach combined with tabulated airfoil data. For each blade, the body forces are distributed radially along a line representing the blade of the wind turbine. At each point of the line, the force is smeared among neighboring nodes with a three-dimensional Gaussian distribution in order to avoid the numerical singular behavior and mimic the chord-wise pressure distribution. The ACL method is implemented into the EllipSys3D code developed by Michelsen (1994) and Sørensen (1995). The EllipSys3D code is based on a multiblock/cell-centered fourth order finite volume discretization of the incompressible Navier–Stokes equations. The code is formulated in primitive variables, *i.e.* in pressure and velocity variables, in a collocated storage arrangement. Rhie/Chow interpolation is used to avoid odd/even pressure decoupling. The actuator line method was implemented in the EllipSys code by Mikkelsen (2003) and for more details we refer to this work or Trolborg

(2008). In the present work the geometry of the Tjæreborg turbine is used as input to generate the wake on which the stability analysis is performed.

3. Stability analysis

The aim of the stability analysis is to study which modes are present and to what extent they grow in order to quantify frequencies leading to a breakdown of the vortex spirals. Figure 1 illustrates the concept of introducing a small sinusoidal perturbation on the tip vortices close to the tip of the blade by adding a time dependent body force.

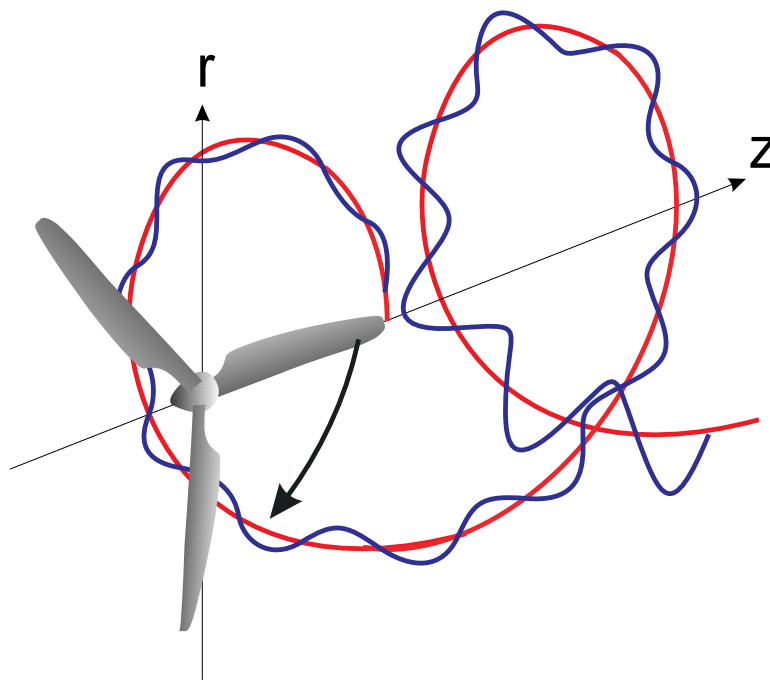


Figure 1. Conceptual sketch of the sinusoidal perturbation introduced close behind the turbine blade. For simplicity, only one spiral is shown. Reproduced from Ivanell et al. (2010).

The perturbation is introduced in axial direction only. The disturbance is positioned close behind the tip and results in a spatially developing disturbance wave on the spiral vortices. This is in contrast to most of previous instability analysis of spiral vortices which calculate the unstable frequencies from an eigenvalue problem. By evaluation of a certain number of velocity fields taken equidistantly over one period, the response from a specific perturbation frequency is evaluated using Fourier analysis. First, a steady state solution is found after which a time resolved computation is performed to reach a periodic solution. N fields, equidistantly spaced in time, are then extracted during one period and used in the calculations of the Fourier coefficients. In order to evaluate the growth along the spiral the amplitude of the perturbation is needed along the vortex spiral. That is determined by identifying the maximum response of the perturbation frequency at each z -position, i.e. at each position in the flow direction.

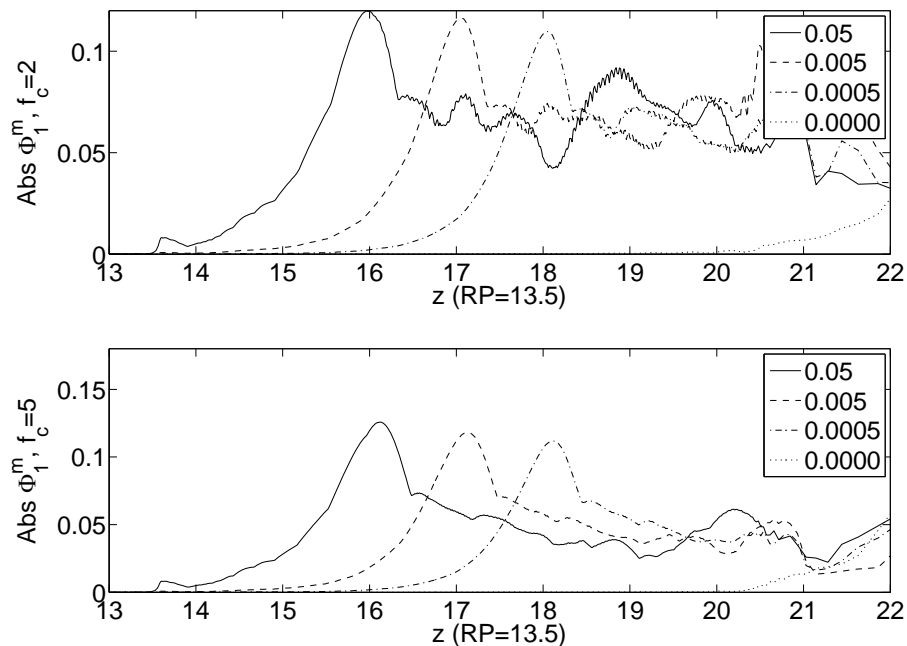


Figure 2. Normalized values of amplitude as function of axial position, with $f_c = 2$ and $f_c = 5$ as perturbing frequency. The perturbed amplitude varies between 0.05 and 0.0005. The figure is reproduced from Ivanell et al. (2010).

Figures 2 and 3 show the development of the amplitude of the first harmonic due to disturbances of dimensionless frequencies $f_c = 2$ and $f_c = 5$ using different perturbation amplitudes. In the figures the abscissa is the axial direction made dimensionless with the radius of the rotor, with the rotor located at $z/R=13.5$. The ordinate shows the amplification of the perturbations which initially is given in a range from 0 to 0.05. Remark that even a zero initial perturbation results in the development of an instability. This is due to numerical truncation error, which by itself introduces the perturbations. Figure 2 illustrates the results on a linear scale, while figure 3 shows the results on a logarithmic scale. From the figures it is clearly seen that the amplitudes develop with an exponential growth.

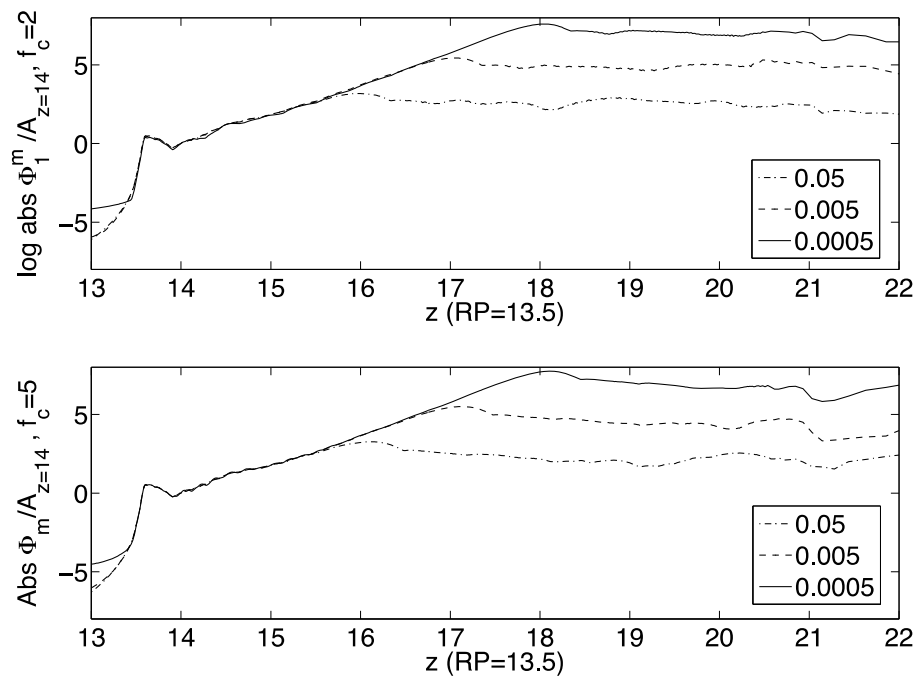


Figure 3. Normalized values of the logarithmic of the amplitude as function of axial position, with $f_c = 2$ and $f_c = 5$ as perturbing frequency. The figure is reproduced from Ivanell et al. (2010).

The data in Figure 3 are normalized by the amplitude at $z = 14$ to compare the growth from the computations with different amplitudes. The result shows that the growth rate is independent of the amplitude of the perturbation since the slope of the data from all perturbation amplitudes are equal up to the point where non-linear effects start to become important. The data from all perturbation amplitudes exhibit an exponential development until they reach a specific value of about 0.1 m/s (see Figure 2). This can be considered as the starting point of non-linear development of the perturbations, which in the following is taken as the point where the tip vortices break down into small-scale turbulence. When choosing smaller amplitudes, the linear part of the extracted signal extends further downstream, since it takes longer time for the instabilities to grow to the extent that they reach a non-linear state. There is, however, a limit to how small the amplitude can be. When the amplitude becomes too small, truncation errors will become of the same order as the disturbance amplitude and will trigger instabilities without applying any perturbation. The dotted curve in Figure 2 illustrates the growth when no perturbation is applied. As a result, that curve gives the lower limit of the disturbance input.

4. Determination the near wake length

By defining the initial length of the near wake behind a rotor as the axial distance from the rotor plane to the position where the tip vortices break down into small-scale turbulence, it is possible to formulate an analytical relationship between this length and the operating conditions of the rotor using the results from the stability analysis. From linear stability theory, assuming exponential growth, we obtain that the amplitude amplification is given as

$$A(t) = A_0 e^{\bar{\sigma} z}, \quad (1)$$

where A_0 is the initial perturbation at location $z = z_0$, A is the amplitude at position z , and $\tilde{\sigma}$ is the dimensional growth rate of the pairing instability. Assuming that the vortices propagate downstream with a constant velocity U_c , one can establish a relation between spatial and temporal growth according to

$$A(t) = A_0 e^{\tilde{\sigma} U_c t}, \quad (2)$$

where t denotes the time, $z = U_c t$. As shown by Lamb (1932) (see also Leweke et al., 2013) the maximum non-dimensional spatial growth rate is given by the universal expression

$$\sigma = \tilde{\sigma} \frac{2h^2 U_c}{\Gamma} = \frac{\pi}{2}, \quad (3)$$

where h denotes the helical pitch of the tip vortex, which corresponds to the axial distance moved by the tip vortex during one turn. Rearranging eq. (3), we get

$$\tilde{\sigma} = \frac{\pi \Gamma}{4h^2 U_c}. \quad (4)$$

This expression is related only to the parameters of the vortices in the wake, e.g. their strength and mutual distance. What is required, however, are the parameters associated with the operational conditions of the wind turbine rotor. In order to establish these relations, we exploit some basic results from momentum theory. Assuming that the wake essentially consists of a system of tip vortices of strength Γ and a root vortex of strength $-N_b \Gamma$, where N_b denotes the number of blades, the following geometrical relationship can be established

$$\frac{N_b h}{2\pi R} = \frac{U_c}{\Omega R} \quad (5)$$

where ΩR is the tip speed of the rotor. From this we get

$$h = \frac{2\pi R U_c}{N_b \lambda U_0}, \quad (6)$$

where λ is the tip speed ratio. Assuming that the rotor is loaded with a constant circulation and neglecting the influence of nonlinear rotational terms, we obtain the following approximate expression for the thrust (see Sørensen & van Kuik 2011),

$$T = \frac{1}{2} \rho R^2 \Omega N_b \Gamma. \quad (7)$$

The assumption of a constantly loaded rotor is of course a relatively crude approximation, but is inspired from the rotor model of Joukowsky, which essentially consist of a root vortex and

distinct tip vortices. The roll-up process of the wake, on the other hand, tends in all cases to form distinct tip vortices. Therefore, the assumption of a constant loaded rotor is consistent with employing a stability analysis of helical tip vortices for developing a criterion for their breakdown. Introducing further the thrust coefficient,

$$C_T = \frac{T}{\frac{1}{2}\rho\pi R^2 U_0^2}, \quad (8)$$

we get

$$\Gamma = \frac{\pi U_0^2 C_T}{\Omega N_b}. \quad (9)$$

Inserting equations (6) and (9) into equation (4), we get

$$\sigma = \frac{N_b C_T \lambda}{16 R \tilde{U}_c^3}, \quad (10)$$

where $\tilde{U}_c = U_c / U_0$. Combining equation (10) with equation (1) results in

$$A(z) = A_0 \exp \left[\frac{N_b C_T \lambda}{\Omega N_b U_c^3} \left(\frac{z}{R} \right) \right]. \quad (11)$$

Taking the logarithmic on both sides and rearranging the equation, we get

$$\frac{z}{R} = \frac{16 \tilde{U}_c^3}{N_b \lambda C_T} \log \left(\frac{A(z)}{A_0} \right). \quad (12)$$

From the previous work by Ivanell et al. (2010) (see figure 3) it was found that the nonlinear breakdown process starts when the amplitude amplification reaches the ratio between the original perturbation and the undisturbed wind velocity, i.e. when

$$\log \left(\frac{A_{\max}(t)}{A_0} \right) \cong -\log \left(\frac{u'}{U} \right). \quad (13)$$

Furthermore, assuming that the turbulence intensity, Ti , is proportional to the perturbation, we get

$$\frac{u'}{U} = C_1 \cdot Ti, \quad (14)$$

Where C_1 is proportionality constant. Combining eqs. (12) – (14), we get the following expression for the position of breakdown of the tip vortices:

$$\left(\frac{l}{R}\right)_{breakdown} = -\frac{16\tilde{U}_c^3}{N_b\lambda C_T} \log(C_1 T_i). \quad (15)$$

It should be noted that the minus sign is due to the fact that the turbulence intensity always is less than one and the logarithmic as a consequence is negative. To the first approximation the propagation velocity of the vortices are given by the free stream velocity, and within this approximation arrive at the final expression for the breakdown position in terms of wind turbine properties. A more accurate propagation velocity can found by exploiting the so-called roller-bearing analogy, in which it is assumed that the vortices move with the average velocity between the wake velocity and the undisturbed wind speed (see e.g. Okulov and Sørensen, 2007). However, it may turn out that the convection velocity is somewhat higher than the one indicated by the roller-bearing analogy and lower than the free-stream velocity. We therefore introduce the following general expression for the convection velocity,

$$U_c = C_2 U_{wake} + (1 - C_2) U_0, \quad (16)$$

where U_{wake} is the wake velocity and $C_2 \in [0,1]$ is a constant that needs to be calibrated against measurements. If $C_2 = 0$ the vortices are convected with free stream velocity whereas $C_2 = 0.5$ corresponds to the roller-bearing analogy. From axial momentum theory we have

$$U_{wake} = U_0 \sqrt{1 - C_T}, \quad (17)$$

which, inserted in to eq. (16), results in the following expression for the convective velocity

$$\tilde{U}_c = 1 + C_2 \left[\sqrt{1 - C_T} - 1 \right]. \quad (18)$$

Introducing this into equation (15), we arrive at the final expression for the position where the tip vortices break down

$$\left(\frac{l}{R}\right)_{breakdown} = -\frac{16 \left[1 + C_2 \left(\sqrt{1 - C_T} - 1 \right) \right]^3}{N_b \lambda C_T} \log(C_1 T_i). \quad (19)$$

This expression gives a measure of the position where the helical tip vortices break down as a function of the intensity of the ambient turbulence level, T_i , and of parameters depending uniquely on the turbine's operational characteristics. In order to evaluate the unknown parameters (C_1 and C_2), numerical simulations are conducted on a small-scale wind turbine using different intensities of the inflow turbulence. The modeled turbine (Adaramola and Krogstad, 2011) consists of a three-bladed horizontal axis rotor, equipped with 14% thick NREL S826 airfoils along the span, which is operated at an effective sectional Reynolds number around $Re = 10^5$. We consider an inflow condition of $U = 10$ m/s and let the rotor operate at optimum performance, corresponding to a tip speed ratio of 6, with $C_T = 0.762$ and $C_p = 0.48$. In the computations the following inflow turbulence intensities are considered:

$T_i = 0.2\%$, 3.0% , and 8.8% , resulting in the following breakdown positions: $z/R = 3.4$, 2.1 , and 1.6 . Using equation (19), we then get $C_1=0.33$ and $C_2=0.52$, corresponding to $U_c=0.73$.

We define the full length of the near wake as the distance from the rotor plane to where a fully developed Gaussian velocity profile is attained. Thus, the total near wake length is here given as the distance from the rotor plane to the breakdown point (eq. 19) plus the distance from this point to where a Gaussian wake deficit is first observed. If we assume that the latter is only affected by the incoming turbulence intensity, we can add an extra term to equation (19), which is assumed to depend directly on the logarithmic of the turbulence intensity. We therefore arrive at the following expression for the total length of the near wake:

$$\left(\frac{l}{R}\right)_{\text{near wake}} = - \left(\frac{16 \left[1 + C_2 \left(\sqrt{1 - C_T} - 1 \right) \right]^3}{N_b \lambda C_T} + C_3 \right) \log(C_1 T_i). \quad (20)$$

The unknown parameter of C_3 can also be obtained by observing that a Gaussian shape for the three different turbulence intensities appears at $z/R = 25.5$, 19.6 , and 12.6 , respectively. This results in an approximate value $C_3 \approx -3$. By substituting the obtained values of the parameters (C_1 , C_2 and C_3) into equation (20), we get the following simple equation for determining the length of the near wake,

$$\left(\frac{l}{R}\right)_{\text{near wake}} = - \left(\frac{6.22}{N_b \lambda C_T} + 3 \right) \log\left(\frac{T_i}{3}\right). \quad (21)$$

Conclusions

Stability theory has been employed to determine the length of the near-wake behind a wind turbine. The stability properties of the wake are determined from a numerical study of the flow around the Tjaereborg wind turbine. The numerical model is based on large-eddy simulations (LES) of the Navier–Stokes equations using the actuator line (ACL) method. The wake is perturbed by applying stochastic or harmonic excitations in the neighborhood of the tips of the blades. It is found that the amplification of specific waves (traveling structures) along the spiral is responsible for triggering the instability leading to wake breakdown. The presence of unstable modes in the wake is related to the mutual inductance (vortex pairing) instability where there is an out-of-phase relationship between the waves on consecutive spirals. Using the non-dimensional growth rate, it was found that the pairing instability has a universal growth rate equal to $\pi/2$. Using this relationship, and the assumption that breakdown to turbulence occurs once a vortex has experienced sufficient growth, an analytical relationship between the turbulence intensity and the stable wake length was derived. The analysis leads to a simple expression for determining the length of the near wake. The expression shows that the near wake length is inversely proportional to the thrust, tip speed ratio and the logarithmic of the turbulence intensity.

Acknowledgment

The work has been carried out with the support of the Danish Council for Strategic Research for the project Center for Computational Wind Turbine Aerodynamics and Atmospheric Turbulence (grant 2104-09-067216/DSF) (COMWIND: <http://www.comwind.org>) and the

Nordic Consortium on Optimization and Control of Wind Farms. Computer time was granted by The Swedish National Infrastructure for computing (SNIC).

References

- [1] Alfredsson, P. H. & Dahlberg, J. A. 1979 A preliminary wind tunnel study of windmill wake dispersion in various flow conditions. *Technical note AU-1499, part 7*.
- [2] Bhagwat, M. J. & Leishman, J. G. 2001 Stability, consistency and convergence of numerical algorithms for time-marching free-vortex wake analysis. *J. Am. Helicopter Soc.* **46** (1), 59–70.
- [3] Felli, M., Camussi, R. & Di Felice, F. 2011 Mechanisms of evolution of the propeller wake in the transition and far fields. *J. Fluid Mech.* **682** (1), 5–53.
- [4] Gupta, B. P. & Loewy, R. G. 1974 Theoretical analysis of the aerodynamic stability of multiple, interdigitated helical vortices. *AIAA J.* **12** (10), 1381–1387.
- [5] Ivanell, S., Mikkelsen, R., Sørensen, J. N. & Henningson, D. S. 2010 Stability analysis of the tip vortices of a wind turbine. *Wind Energy* **13** (8), 705–715.
- [6] Lamb, H. 1932 *Hydrodynamics*, 6th edn. Cambridge University Press.
- [7] Leishman, G., Bhagwat, M. J. & Ananthan, S. 2004 The vortex ring state as a spatially and temporally developing wake instability. *J. Am. Helicopter Soc.* **49** (2), 160–175.
- [8] Leweke, T., Bolnot, H., Quaranta, U. & Le Dizès, S. 2013 Local and global pairing in helical vortex systems. Int. Conf. on Aerodynamics of Offshore Wind Energy Systems and Wakes, Lyngby, Denmark.
- [9] Michelsen, J. A. 1994 Block structured multigrid solution of 2D and 3D elliptic PDE's. *Tech. Rep.* AFM 94-06. Dept. of Fluid Mech., Technical University of Denmark, DTU.
- [10] Mikkelsen, R.F. 2003 Actuator disc methods applied to wind turbines. PhD thesis, Dept. of Fluid Mech., Technical University of Denmark, DTU.
- [11] Okulov, V. L. & Sørensen, J. N. 2007 Stability of helical tip vortices in a rotor far wake. *J. Fluid Mech.* **576** (1), 1–25.
- [12] Sørensen, J. N. 2011 Instability of helical tip vortices in rotor wakes. *J. Fluid Mech.* **682** (1), 1–4.
- [13] Sørensen, J. N. & van Kuik, G. A. M. 2011 General momentum theory for wind turbines at low tip speed ratios. *Wind Energy* **14** (7), 821–839.
- [14] Sørensen, J. N. & Shen, W. Z. 2002 Numerical modeling of wind turbine wakes. *J. Fluid Eng.* **124** (2), 393–399.
- [15] Sørensen, N. N. 1995 General purpose flow solver applied to flow over hills. PhD thesis, Riso National Laboratory, Roskilde.
- [16] Troldborg, N. 2008 Actuator line modeling of wind turbine wakes. PhD thesis, Dept. of Fluid Mechanics, Technical University of Denmark, DTU.
- [17] Widnall, S. E. 1972 The stability of a helical vortex filament. *J. Fluid Mech.* **4** (54), 641–663.
- [18] Adaramola, M. and Krogstad, P.Å. 2011 Experimental investigation of wake effects on wind turbine performance. *Renewable Energy* **36**, 2078–2086